Sheet Metal Unfold Rule Equations

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1 Introduction

1.1 Bend Parameters Used in this Document

The following parameters and terms are used throughout this document.

![Figure 1 – Basic Sheet Metal Bend Parameters](image)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Thickness</td>
<td>Thickness of the Sheet Metal Part</td>
</tr>
<tr>
<td>Ri</td>
<td>Inside Radius</td>
<td>Inside Radius of a Bend</td>
</tr>
<tr>
<td>t</td>
<td>Neutral Line Position</td>
<td>The distance from the Inside bend to the Neutral Line, the line along which no deformation occurs during the bending process</td>
</tr>
<tr>
<td>Rn</td>
<td>Neutral Radius</td>
<td>Radius of Neutral Line, the line along which no deformation occurs during the bending process</td>
</tr>
<tr>
<td>Ln</td>
<td>Bend Allowance</td>
<td>Arc Length of the Neutral Line in the bend. Ln does not changed during the bending process, it is constant</td>
</tr>
<tr>
<td>α</td>
<td>Bend Angle (Outside Bend Angle)</td>
<td>The angle of the bend. Equivalent to the outside bend angle as shown in Figure 1, which is the outside angle between the flange and the base plate</td>
</tr>
<tr>
<td>β</td>
<td>Opening Angle (Inside Bend Angle)</td>
<td>The complement of α, the inside angle between the flange and the base plate.</td>
</tr>
<tr>
<td>SBout</td>
<td>Outside Set Back</td>
<td>The distance from the bend line to the virtual sharp intersection of the outside flange faces. Typically only valuable for α ≤ 90°.</td>
</tr>
</tbody>
</table>
### 1.2 Mathematical Definitions for Bend Parameters

Given the definitions above, there are a few basic mathematical relationships that can be expressed and are useful in this document.

\[
t = R_n - R_i 
\]  \hspace{1cm} (1)

\[
\beta = 180^\circ - \alpha 
\]  \hspace{1cm} (2)

\[
SB_{out} = \tan\left(\frac{\alpha}{2}\right) \cdot (R_i + T) 
\]  \hspace{1cm} (3)

\[
SB_{in} = \tan\left(\frac{\alpha}{2}\right) \cdot R_i 
\]  \hspace{1cm} (4)

### 2 Calculating Lengths of Unfolded Bends

#### 2.1 Bend Allowance

The most basic way to calculate the length of an unfolded part is by taking the length of a line in the bend that does not change during the bend process. This is known as the Neutral Line or Neutral Axis; and it is the line along which no strain in the material has taken place.
The goal is to get the total length \( L \) of a workpiece as shown in Figure 2:

\[
L = L_1 + L_2 + L_n \tag{6}
\]

\( L_n \) is known as the Bend Allowance. Since it is the arc length of the Neutral Line, it can be calculated using a simple equation:

\[
L_n = \alpha \cdot \frac{\pi}{180} \cdot R_n = \alpha \cdot \frac{\pi}{180} \cdot (R_i + t) \tag{7}
\]

where \( \alpha \) is in Radians. The Neutral Line is located at a distance \( t \) from the inside of the bend radius. This distance is determined experimentally, and is usually expressed as a percentage of the total thickness \( T \):

\[
t = K \cdot T \tag{8}
\]

Where typically:

\[
0 \leq K \leq 1 \tag{8a}
\]

**Note:** \( K \) can be less than 0 or greater than 1 in theory and even in practice, but in general it is expected to be between 0 and 1.

This percentage \( K \) is known as the **K-Factor**, and is determined in a variety of ways (which is discussed later). Given these, the bend allowance can always be calculated by the following:

\[
L_n = \alpha \cdot \frac{\pi}{180} \cdot (R_i + K \cdot T) \tag{9}
\]

### 2.2 Bend Deduction

The beauty of the Bend Allowance method is that it can be calculated using a provided K factor and the known properties of the bend. It is simple and straight forward. Getting the K factor to begin with is another problem.

Several rule of thumb and standardized K factors exist, but in general K factors are determined experimentally. K factors express the amount of stretch when bending sheet metal. The actual amount of stretch is dependent on several factors:

- **Material Properties**
  - Material Thickness
  - Strength of material
  - Material Hardness
  - etc.
- **Tooling and bend properties**
  - Angle of the Bend
  - Radius of the Bend
  - Type of Bend operation
  - etc.

This process to calculate K Factors requires taking blank sheets of known lengths, folding them up, and measuring the lengths of the two sides of the bend.
Measuring is the real trick. Bending a known length of sheet metal is easy, but to calculate the Bend Allowance (and therefore the K Factor), the lengths of $L_1$ and $L_2$ must be measured. This is not easy since the exact location of the bend line is really not known.

It is much easier to measure to the virtual intersection (or Virtual Sharp) of the flanges:

Figure 3 – Measuring the flange lengths up to a bend is not practical in practice

Figure 4 – Measuring experimental parts using their “Virtual Sharps”

Figure 5 – Parameters used to determine Bend Deductions
What a person can reliably measure is shown in Figure 5, which are the lengths $L_a$ and $L_b$. Experimentally the user knows the initial length of the test piece ($L$), and it is known that the outside of the metal is stretching, so the following can be deduced:

$$L = L_a + L_b - L_{BD} \tag{10}$$

where $L_{BD}$ is known as the **Bend Deduction**, and $L_a$ and $L_b$ are not linear values:

$$L_a = L_1 + L_{SetBack} \tag{11a}$$
$$L_b = L_2 + L_{SetBack} \tag{11b}$$

where:

$$L_{SetBack} = \tan \left( \frac{\alpha}{2} \cdot (R_i + T) \right), \quad 0^\circ \leq \alpha \leq 90^\circ$$
$$L_{SetBack} = (R_i + T), \quad 90^\circ < \alpha \leq 180^\circ \tag{11c}$$

### 2.3 Relationship between Bend Deduction, Bend Allowance, and K Factor

Given that $L$ must be the same no matter how it is calculated, equation 10 and 6 are equal. This means that a relationship between the two can be determined:

$$L_1 + L_2 + L_n = L_a + L_b - L_{BD} \tag{12a}$$
$$L_1 + L_2 + L_n = L_1 + L_{SetBack} + L_2 + L_{SetBack} - L_{BD} \tag{12b}$$
$$L_n = 2 \cdot L_{SetBack} - L_{BD} \tag{12c}$$

This also means that the K factor can be calculated from the Bend Deduction:

$$K = \frac{180^\circ}{\alpha \cdot \pi} \cdot \left( 2 \cdot L_{SetBack} - L_{BD} \right) - R_i \tag{14}$$

### 2.4 Notes on Use of Terminology

Many people use the terms **Bend Allowance** and **Bend Deduction** interchangeably. It is even common to see charts on a shop floor titled “Bend Allowance”, and the chart is actually a chart of Bend Deductions. However the terms do not mean the same thing (as outlined above).

Another common mix up in terminology is the difference between the **Bend Angle** ($\alpha$), and the **Opening Angle** ($\beta$). This can lead to a lot of confusion and errors when designing and calculating flat patterns.

One last note on terminology: some people use the term **Bend Compensation**. Bend Compensation is the negative value of the Bend Deduction:

$$L_{BC} = -L_{BD} \tag{15}$$
3 Common Unfold Rules

Several rules for unfolding exist; either found in standards and other sources. For truly accurate unfolding, manufacturers generate their own. This section covers known rules in industry and common practices known to exist.

3.1 Lockheed Bend Allowance & K Factor

K Factors were first developed by Lockheed Martin in the 1950’s \(^1\). Typically K Factors are a single numeric value used for either all material/thickness/angle/radius combinations, or for specific sets. This makes for a very linear, simple to use formula with Bend Allowance.

By default Inventor uses a K factor of \( K = 0.44 \). This K factor is derived from the Bend Allowance formula developed by Lockheed:

\[
BA_{Lockheed} = \alpha \cdot (0.017453 \cdot R_i + 0.0078 \cdot T) \quad (16a)
\]

\[
\alpha \cdot (0.017453 \cdot R_i + 0.0078 \cdot T) = \alpha \cdot \frac{\pi}{180^\circ} \cdot (R_i + K \cdot T) \quad (16b)
\]

\[
K = 0.0078 \times \frac{180^\circ}{\pi} = 0.44 \quad (16c)
\]

Note: Most likely this formula was developed for a class of materials, probably Cold Rolled Steels and Spring Steels. Specifics were not known by the author at the time of writing.

3.2 Machinery’s Handbook Bend Allowances

Machinery’s Handbook offers three bend allowance formulae for various materials \(^2\). The formulae are for bends of 90º.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Bend Allowance Formula*</th>
<th>Derived K Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Brass, Soft Copper</td>
<td>( L_n = (0.55 \cdot T) + (1.57 \cdot R_i) ) (17a)</td>
<td>( K = 0.35 )</td>
</tr>
<tr>
<td>Half-Hard Copper &amp; Brass, Soft Steel, Aluminum</td>
<td>( L_n = (0.64 \cdot T) + (1.57 \cdot R_i) ) (17b)</td>
<td>( K = 0.407 )</td>
</tr>
<tr>
<td>Bronze, Hard Copper, Cold-Rolled Steel, Spring Steel</td>
<td>( L_n = (0.71 \cdot T) + (1.57 \cdot R_i) ) (17c)</td>
<td>( K = 0.452 )</td>
</tr>
</tbody>
</table>

*Note: for angles other than 90º, multiply the Bend Allowance \( L_n \) by the Bend Angle \( \alpha \) in degrees divided by 90:

\[
L_n (\neq 90^\circ) = L_n \cdot \frac{\alpha}{90} \quad (17d)
\]

\(^1\) “Bend deduction charts, Where can I get one?”, Steve D. Benson, July 26, 2001, (http://www.thefabricator.com/Bending/Bending_Article.cfm?ID=356)

3.3 DIN 6935

The DIN 6935\(^3\) standard outlines several practices and standards for creating bent sheet metal parts. This includes the development of a set of Bend Compensation based formulae for calculating the unfold length of bends:

\[
L = L_a + L_b + V \tag{18}
\]

where \(V\) is the Bend Compensation. Note that it is the negative of the Bend Deduction (as described in Section 2.4).

\[
V = -L_{BD} \tag{19}
\]

The Bend Compensation \(V\) is determined based on the **Opening Angle** (\(\beta\)), as shown below:

Equation (20a), (20b), & (20c)

<table>
<thead>
<tr>
<th>Bend Compensation Formula</th>
<th>Valid (\beta) Range</th>
</tr>
</thead>
</table>
| \[
V = \pi \cdot \left(\frac{180^\circ - \beta}{180^\circ}\right) \cdot \left(R_i + \frac{T}{2} \cdot K_{DIN}\right) - 2 \cdot (R_i + T)
\] | \(0^\circ \leq \beta \leq 90^\circ\) |

| \[
V = \pi \cdot \left(\frac{180^\circ - \beta}{180^\circ}\right) \cdot \left(R_i + \frac{T}{2} \cdot K_{DIN}\right) - 2 \cdot (R_i + T) \cdot \tan\left(\frac{180^\circ - \beta}{2}\right)
\] | \(90^\circ < \beta \leq 165^\circ\) |

| \(V = 0\) | \(165^\circ < \beta \leq 180^\circ\) |

where:

\[
K_{DIN} = \begin{cases} 
0.65 + 0.5 \cdot \log \left(\frac{R_i}{T}\right), & 0.05 \leq \frac{R_i}{T} \leq 5 \\
1, & 5 < \frac{R_i}{T}
\end{cases}
\tag{21}
\]

**Note:** The value of \(K_{DIN}\) is not the same as a K Factor as described in Section 2.1, and should not be confused with standard K Factors.

\(^3\) Formula and data from DIN 6935, Deutsches Institut für Normung, October, 1975
3.4 Bend Deduction Tables

While K Factors are convenient and tend to be simple to use (especially for CAD software), for the most accurate development of flat patterns, Bend Deduction tables, based on actual measured data, are the preferred method.

In a given manufacturing environment typically there are a finite set of materials, gauges (thicknesses), bending processes, and tooling. Because of this finite limit, generation of Bend Deduction Tables for all of the possible combinations of these is practical.

Typically these Bend Deduction tables are created using the method described in section 2.2, where samples of known length are bent and measured, and the Bend Deduction is calculated.

Historically K factors are not generated from these tables. The reason is simple: when manually drafting or drafting in 2D such as in AutoCAD to create a flat pattern drawing, using a Bend Deduction Table is simple and straightforward. All the designer needs is the dimensions of the flanges to their virtual sharps (common drawing practice), and the appropriate parameters to look up the Bend Deduction. Only simple addition is needed to calculate the final length to draw.

Most Bend Deduction tables are generated only for 90° bends. For bends other than 90°, some other provisions are used. Some companies will generate tables for all the possible angle/radius/thickness combinations.

![Figure 6 – Example of a Detailed Bend Deduction Table](image)

Note: The table pictured in Figure 6 goes from 5°~170° in 1° increments, and covers 18 different Thicknesses and 13 different Radii. Only 3 of the 160+ pages are shown.

Since so many Bend Deduction tables exist, Inventor supports the use of Bend Deduction Tables as an alternative to K Factor.

It is worth noting that in older versions of Inventor, the format supported by Inventor assumes that the Bend Deduction Table is using the Opening Angle ($\beta$), and not the Bend Angle ($\alpha$). Inventor features are designated using the Bend Angle ($\alpha$), however. Current versions of Inventor you can specify if the bend table is using Opening or Bend Angles.
Figure 7 – Inventor Bend Deduction Tables